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Note on the Settling of Small Particles in a Recirculating Flow

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IN the formation of platelet aggregates or microemboli in flows containing a vortex-like separated region, the residence time of a platelet (or other small particle) within the separated part of the flow is of prime importance. One factor which could limit the residence time is the tendency of small particles to settle under the influence of gravity. In connection with this problem, we have evaluated a more general situation: the kinematics of the settling of a small particle in any recirculating flow characterized by two-dimensional closed streamlines in a vertical plane. The particle is assumed to have constant settling velocity corresponding to a small sphere moving under the influence of gravity at terminal velocity, and to have $v_s \ll u$, where v_s is the magnitude of the settling velocity and u that of the flow. For this situation, one might expect that the vertical distance displaced during a complete circulation to be simply the product of the settling velocity and the circulation time. The calculation presented here shows that this is not so, and that to a good approximation, the particle returns to its original streamline in a complete circulation, and the net vertical distance displaced is zero.

The particle motion is taken to be that of the background flow with the added constant settling velocity. The particle starts at some point on a streamline ψ_1 and moves to adjacent streamlines due to the settling motion in the negative y direction. (The x-axis is horizontal.)

Let ds be a small length along ψ_1. In traversing ds, the particle will settle a distance dy below ψ_1 given by

$$dy = -v_s(ds/u), \quad dy \ll ds \tag{1}$$

corresponding to a change in stream function

$$d\psi = (\partial\psi/\partial y)dy = u_x dy = -u_x(v_s ds/u) = -v_s(u_x/u\, ds) \tag{2}$$

or

$$d\psi = -v_s dx$$

Thus the net change in ψ during the flow with a constant v_s, is simply

$$\psi_2 - \psi_1 = -v_s(x_2 - x_1) \tag{3}$$

The result, Eq. (3), is valid to first order in v_s/u and is independent of the details of the flow. It states that when $x_2 = x_1$ then $\psi_2 = \psi_1$, and hence a particle returns to its original streamline twice in a complete circulation, and does *not* settle out of the flow. A particle "falls" inward during one part of the flow and outward during the other, the two excursions completely cancelling; independent of the details of the flow. A typical trajectory is shown in Fig. 1. (The points P and Q in the figure have the same x coordinate and indicate positions where the particle trajectory crosses the streamline ψ_1.)

The result, Eq. (3), also implies that, to first order in (v_s/u), the displacement δ normal to an initial streamline is:

$$\delta = (v_s/u)\Delta x \tag{4}$$

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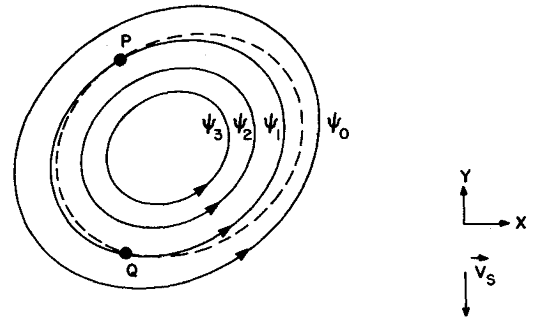

Fig. 1 Trajectory (dotted line) of a particle as it "settles" in a recirculating flow.

since $\Delta\psi = u\delta$. Thus for situations involving a vortex-like region surrounded by boundaries, (such as walls, or dividing streamlines in a separated flow) settling can affect only those particles which lie on streamlines which pass within δ of a boundary as given by Eq. (4), with Δx determined by the projection of the streamline path on the x axis. Since we are taking $v_s \ll u$, the streamlines affected represent only a small part of the flowfield. Thus, to first order in (v_s/u), settling due to gravity should be unimportant for particles in any two-dimensional recirculating flow in a vertical plane.

Localized Diamond-Shaped Buckling Patterns of Axially Compressed Cylindrical Shells

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Introduction

THE upper stability limit of cylindrical shells with classical simply supported boundary conditions was treated long ago by Flügge[1] and Pflüger.[2] This upper limit, as is well known, fails to serve as a design criterion due to the snap-through effect (Durchschlag) common in shell structures. An explanation of this snap-through characteristic and an approximate solution of this problem has been given by von Kármán et al.[3] This work which is now a classic, has been developed by many authors[2,4,5] and has also been further refined and improved in recent times, notably by Esslinger.[4] Because of the mathematical difficulty and the large algebraic labor involved, the majority of these works have dealt with simply supported boundary conditions and an over-all buckling pattern to keep the amount of calculation in reasonable limits. Indeed, the only exception known to the author is that of Ref. 5 where, unfortunately, no results were obtained because of the convergence difficulties in the large computer program.

In the following we have, therefore, the rather humble aim of obtaining only an estimation of the lower stability limit of the free edge cylinder. The method is based on a simple differential equation with Pogorelov coefficients.[6] Mechanically, the method can be interpreted as the buckling of a strut on a "bending" foundation with an attenuated eigenvector. With "bending," it is meant that the elastic constant of this fictitious foundation is due to the isometric bending in the circumferential direction.[9]

Energy Functional and the Isometric Transformation

In general, a shell surface is a two-dimensional Riemann space for which the integrability conditions are given by the

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